The application of the mathematical model (MODFLOW) to simulate the behavior of groundwater flow in Umm Er Radhuma unconfined aquifer

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Abstract:

The MODFLOW model is applied to evaluate groundwater availability and predict water levels in response to increasing pumping. A transient simulation of groundwater regime for Umm Er Radhuma unconfined aquifer using a mathematical model that assumes an initial draft rate condition of 250 m$^3$/day per well increasing herein gradually by adding 50 wells a year has revealed an aquifer head loss reaches up to 5.4m which may cause dryness in the upper part of the aquifer in the year 2017 at certain locations, especially, in the west and the north west side.

The model grid consist of (43) columns and (43) rows and the cell dimension are 2500 by 2500 m; the total number of active cells within the study area are (530) cells. A steady-state model was developed on the basis of average groundwater heads for a 10 years period (1979 through 1988). The transient simulations were conducted using monthly averages for recharge and discharge from pumping data of the drilled wells. Good agreement was found between measured and calculated hydraulic heads at the aquifer during different time steps and time series of water levels in the domain. The percent of discrepancy was 0.68%.

Introduction:

Umm Er Radhuma unconfined aquifer is an important aquifer in the Western Desert. It extends about (60,068) Km$^2$ in the Iraqi Western Desert covering the area of Umm Er Radhuma unconfined aquifer inside Iraq near the Iraq-Saudi Arabia borders Figure (1). The usage of the application of MODFLOW mathematical model for the description and prediction of groundwater system behavior has increased significantly over the last few years. The software of this model has been used to fit Umm Er Radhuma unconfined aquifer condition in order to simulate its behaviors under several assumptions for any desired duration. The version groundwater-flow numerical model (Processing MODFLOW for windows PMWIN PRO V.7, by
Chiang and Kinzelbach) has been used and developed to evaluate the availability of groundwater and predict water levels in relation to the increase of pumping during the periods 2001 through 2010. First, a steady-state model was developed on the basis of average groundwater heads for 10 years period (1979 through 1988). Then, transient simulations were conducted using monthly groundwater recharge and discharge.

Figure (1) Location map

Geologically Umm Er Radhuma formation in the Western Desert has been recently sub-divided into two members. The first member (Lower member of Middle Paleocene) can further be sub-divided into two rock units according to the lithological and physical properties: Lower chalky unit and Lower shelly unit.

The second member (Upper member of Upper Paleocene) can be sub-divided into three rock units according to the lithological and physical properties: Shelly - chalky unit, Upper chalky unit and Upper shelly unit Figure (2).

Figure (2) Geological map
Purpose of Study:
The purpose of building a model is to develop useful information about groundwater resources of the unconfined aquifer. Through the simulation of various model conditions by suggesting different pumping dialogues from the aquifer, the resulting aquifer hydraulic head will be noticed through three simulations (Steady, Transient and Predictive simulations), (Harbaugh et.al., 2000).

Method of Study:
1- Hydrogeological data bank was used as the principal data for the model. These data were precisely documented until 1988. Further data for the wells which were drilled in the area in 1999 and 2000 were added to verify the model.
2- GIS (V. 9.1) was used to draw the maps using 3D spatial analysis to feed the model with required data.
3- Processing MODFLOW for Windows {(PMWIN) V 7.0.2} was applied in this study to simulate the behavior of groundwater through this study.

The use of numerical models for the simulation of groundwater flow:
The paragraphs below give a brief definition and background information regarding groundwater modeling. In the most abstracted words, groundwater model is a unification of the concepts of an aquifer that allow hydrogeologists to make conceptual prediction of the aquifer future conditions. It’s usually a computer based representation on the different aspects of a natural hydrogeological system. It consists of two components, a conceptual model, and a graphical presentation of hydrogeological setting and the mathematical model. A valid model should approximate the behavior of the aquifer and provide a tool for prediction and quantification of impacts due to groundwater extraction. Groundwater models are often taken as an integral part of the decision support tools for water resources management. They are increasingly used to predict the impacts of proposed developments and policies concerning water management (Harbaugh et.al., 2000).

The modeling process consists of the initial proposal, the modeling plan, the construction and calibration of the model, the design of the scenario presentation of results and achieving the model. MODFLOW is a finite-difference modeling program, which simulates groundwater flow in three dimensions. The code or computer program is written in FORTRAN 77. The program has a modular format, and consists of a ‘main’ program and a series of highly independent subroutines called ‘modules’. The modules are grouped into ‘packages’. Each package deals with a specific feature of the hydrologic system which is to be simulated, such as flow of rivers or flow into drains, or with a specific method of solving linear equations which describe the flow system. The division of the program into modules facilitates the examination of each hydrologic feature in the model independently. Another advantage of having the modular structure is that new options/features could be added to the program without much change to the existing code. As with most computer programs that have been available for a long time, MODFLOW underwent several version updates. It was originally documented in 1984 by Harbaugh and McDonald. The second version of MODFLOW was documented by McDonald and Harbaugh (1988) and another version is often called MODFLOW-88 to distinguish it from other versions. The third version is called MODFLOW-96 (Harbaugh and McDonald, 1996a and 1996b). The
fourth version is called MODFLOW-2000 (Harbaugh et al. 2000), while the latest version is called MODFLOW-2003. This latest version, i.e. MODFLOW-2003, has the largest number of options/features and thus it has been decided that it would be used for the development of the proposed SDSS (Special Decision Support System). The simultaneous equations used by MODFLOW for each finite difference cell is derived using Darcy's Law and the law of the conservation of mass. This derivation gives a partial differential equation, which is used by MODFLOW. The transient partial-differential equation of groundwater flow used in MODFLOW (Harbaugh and McDonald, 1988) is:

\[
\frac{\partial}{\partial x} \left[ K_x \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_y \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K_z \frac{\partial h}{\partial z} \right] + W = S_s \frac{\partial h}{\partial t} \quad \ldots (1)
\]

where, \( K_x, K_y, \) and \( K_z \) are the values of hydraulic conductivity along the x, y and z coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity.

\( h \) : is the potentiometric aquifer head

\( W \) : is the volumetric flux per unit volume representing sources and/or sinks of water, with \( W<0.0 \) for flow out of the groundwater system, and \( W>0.0 \) for flow in.

\( S_s \) : is the specific storage of the porous material; and

\( t \) : is time.

This equation, when combined with boundary and initial conditions, describes transient three-dimensional groundwater flow in a heterogeneous and anisotropic medium, provided that the principal axes of hydraulic conductivity are aligned with the coordinate directions. McDonald and Harbaugh (1988) used a finite difference version of this equation in MODFLOW, where the groundwater flow system is divided into a grid of cells. For each cell there is a single point called node at which the head is calculated.

Packages which have been included in MODFLOW-88 are the basic package, the block-centered flow package, the well package, the recharge package and the solver packages. These packages have been extended in MODFLOW 2000. In addition to the packages mentioned above, MODFLOW now includes the layer-property flow package, the horizontal flow barrier package, the river package, the drain package, the evapotranspiration package, the general-head boundary package, and the constant-head boundary package (Harbaugh et al., 2000).

A number of changes have been made to USGS modular finite difference groundwater flow model over the past several years. The author of MODFLOW [Harbaugh and McDonald] have collected comments from model users about changes that would make MODFLOW more useful (Harbaugh, et al., 2000).

Processing MODFLOW for Windows (PMWIN) is the best simulation system exists till now. It comes accompanied with a professional graphical preprocessor and postprocessor, 3-D finite-difference groundwater models MODFLOW-88, MODFLOW-96, and MODFLOW 2000; solute transport models MT3D, MT3DMS, RT3D and MOC3D; particle tracking model PMPATH 99; and inverse models UCODE and PEST-ASP for automatic calibration. Now, A 3D visualization and animation package, a 3D Groundwater Explorer are also included (Harbaugh, et al., 2000).
Processing MODFLOW for windows PMWIN PRO V.7 is a complete simulation for groundwater modeling and transport process with the 3 dimensional finite differences. The model simulates flow in 3 dimensions; groundwater flow within the aquifer is simulated using a block centered finite difference approach. Layers can be simulated as confined, unconfined, and a combination of the aquifer types. Flow associates with external stresses such as wells, aerial recharge, evapotranspiration, drains and streams can also be simulated .The finite differences equation can be solved using strongly implicit procedure [PMWIN pro manual 2004]

Application of mathematical model to simulate the Unconfined Aquifer:

Mathematical model is constructed for the simulation of groundwater flow in the unconfined aquifer. An understanding of groundwater flow is necessary to develop an efficient program for the future management. A model was constructed in a stepwise fashion, beginning with a model domain extension. The primary source of groundwater to the model is the recharge, while the primary sinks for groundwater within the model area are the wells. Calibration targets were considered as the hydraulic heads.

In this investigation the model was constructed to simulate the Pliocene unconfined aquifer of Umm Er Radhuma which consists of a single layer, the model domain is discretized into grids; therefore a solution for the flow equation can be calculated at any point in the domain.

Initial Model Development:

After constructing the conceptual model and selecting the modeling software, which in this case is [PMWIN pro], the features of the conceptual model are transferred to an input file that defines the mathematical model. The Boundary condition, grid dimensions and spacing, initial aquifer properties and time steps features are specified as the basic components of the model.

Grid Design:

Model grids discrete the continuous natural system into segments (cells) that allow numerical solution to be calculated. The spacing between nodes which is called grid resolution should be responsive to sharp changes. The overall size of the grid (total number of nodes) should be adequate to define the problem and the results of the procedure consistent with modeling objectives, but not so large to cause excessive preparation and computation requirements. The model grid consists of (43) columns and (43) rows and the cell dimensions are 2500 by 2500 m, the total number of grid cells are (1849) of which (530) are active cells within the modeled area.

Boundary Conditions:

Boundary conditions have a great influence on the computation of flow velocities and heads within the modeled area. Two types of boundary conditions are used for the unconfined Umm Er Radhuma aquifer groundwater flow mode. First, specified head of positive values (+1) at the boundary defining an active cell when expressing the domain inside. Second, a (0) value when defining an inactive cell (or no flow) at any outer flow boundary cell.

Initial Hydraulic Head:

MODFLOW requires initial hydraulic heads at the beginning of a flow simulation. Initial hydraulic heads at constant head cells are used as specified head values of these cells. For steady
state simulation, the initial heads are used as starting values. Actual, confidential head values were derived from the hydrogeologic data bank of the Ministry of Water Resources. Figure (3) shows the isopotential lines of the aquifer. These same values were loaded to the model as initial hydraulic heads.

Figure (3) The data of the initial hydraulic heads for unconfined aquifer that used in the model

Aquifer Material Properties:

The top and bottom elevations of the layer were also loaded; the values at the top of aquifer were derived from Figure (4), While the values at the bottom of the aquifer were derived from Figure (5).

Figure (4) Top of Umm Er Radhuma unconfined aquifer
Transmissivity is one of the input readings required in the model; the distribution of transmissivity values showed in Figure (6) and a value was loaded for each cell.

Flow packages representing recharge by length/time dimension (L/T) which are usually positive values, and discharge (L³/T) with negative values which reflect the pumping from wells. Recharge is defined by assigning the data for each vertical column of cells. The input of recharge- discharge parameters are assumed to be constant during a given stress period.;
MODFLOW assumes that a well penetrates the full thickness of the cell. To calculate heads in each cell in a finite difference grid, PMWIN pro prepares one finite difference equation for each cell; (Processing Modflow pro V.7). A strongly implicit procedure (SIP) package is used to solve the system of finite difference equation.

**Time Stepping:**

Time stepping is the discretion of flow equation through time and is used in the transient simulation, while in the steady state simulation time variation is not included.

**Steady state simulation:**

The steady state flow is the state in which the volume passing a given point per unit of time remains constant. An initial steady-state condition is required for time-dependent modeling of groundwater flow.

After feeding the data and running the model, the hydraulic heads are the primary results of MODFLOW in a steady state simulation as shown in Figure (7).

**Model Calibrations:**

Calibration is the process of adjusting model inputs to achieve a desired degree of correspondence between the model simulation results and the natural groundwater flow system. In other words, calibration continues until a reasonable match is obtained with the measured values (usually hydraulic heads). Thus, model calibration is made in order to reach a matching between input data and model simulation results.

![Figure (7) Calculated heads for Umm Er Radhuma unconfined aquifer using the steady state simulation](image)

Draw down head differences between the initial hydraulic head and the calculated hydraulic head, Figure (8), show a good match with a difference of no more than 1 m.
Figure (8) Draw down (head differences) in the modeled area

The model PMWIN pro calculates water budget for each subregion in each time step, the percent of discrepancy =\[100\times(\text{IN-OUT}) / (\text{IN+OUT}) / 2\] in this step is 0.68 %, this means that the model equation has been reasonably solved.

**Transient Simulation & Model Development:**

The applicability of groundwater model to a real situation depends on the accuracy of the input data. The determination of these requires considerable study, like the collection of hydrological data (rainfall, evapotranspiration, surface runoff and soil moisture). Groundwater recharge evaluation was based on these calculations.

For transient calculations of groundwater flow, a stable initial condition which is created from a steady state simulation has been adopted for this simulation, beside sinks (wells) and sources (recharge) which are introduced.

An initial base line condition was simulated with no presence of pumping wells, while recharge was modeled as a source. Water surplus (Groundwater recharge + Surface runoff) had been calculated previously indicating that the aquifer receives 21.1 mm/year as an average recharge from rainfall. Since the model used \((L/T)\) as units for recharge, the values changed to m/day for the wet months. These values were included in the groundwater model during the development of the transient simulation after being divided among the different wet months as shown in table (1).
Table (1) Monthly recharge values adapted by the model

<table>
<thead>
<tr>
<th>Months</th>
<th>Recharge (mm)</th>
<th>Recharge (m/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct.</td>
<td>0.32</td>
<td>0.0000106</td>
</tr>
<tr>
<td>Nov.</td>
<td>6.19</td>
<td>0.0002063</td>
</tr>
<tr>
<td>Dec.</td>
<td>2.44</td>
<td>0.0000813</td>
</tr>
<tr>
<td>Jan.</td>
<td>4.71</td>
<td>0.000157</td>
</tr>
<tr>
<td>Feb.</td>
<td>2.54</td>
<td>0.0000846</td>
</tr>
<tr>
<td>Mar.</td>
<td>3.29</td>
<td>0.0001096</td>
</tr>
<tr>
<td>Apr.</td>
<td>1.6</td>
<td>0.0000533</td>
</tr>
</tbody>
</table>

Time stepping in transient simulation:

The time of one year simulation, without sinks (wells), is divided into 12 stress periods, each stress period represents a month, while the length of each stress period is divided into days. The total time steps therefore equal 12 months, while the total simulation time equals 3.65E+2 days. The model output in the monthly flow map depends on the variation of the above package (groundwater recharge).

The calculated heads for each time step show slight difference in values. Figure (9) shows the calculated heads for the last step in this simulation month (September) for a dry period.

One of the basic ways to evaluate the quantity of groundwater through an aquifer system is by using the water budget. (Donna, et.al., 1998).

The fundamental equation of a water budget (or water balance) is the sum of inputs minus the sum of outputs equals the change in storage of the system:

\[ \sum \text{Inputs} - \sum \text{Outputs} = \Delta \text{Storage} \]
Water budget calculation by the model is a volumetric water budget for the entire model at the end of each time step. In numerical solution terms, the system of the equations solved by the model actually consists of a flow continuity statement for each model cell. The continuity principle should also be applicable to the total flows of the entire model or subregion. This means that the difference between the total inflow and the total outflow should equal the total change in the storage (Alzubari, W, 2004).

Figure (10) shows the water budget of the entire model through the last time step of this simulation. From the volumetric budget for the entire model it seems that the discrepancy percent is nil.

Another simulation with sinks (wells) is divided into 2 stress periods, representing wet and dry periods. The recharge from rainfall equals (0.0007027) m/d while the discharge from wells is (78523.2) m³/d. The result shows a slight difference in hydraulic heads produced by the simulation of the two stress periods as shown in figure (11).

**Predictive simulation:**

A model may be used to predict future groundwater flow conditions, such simulation estimates the hydraulic response of an aquifer, and also it can predict the pumping rate needed to monitor the hydraulic heads. A pumping strategy is a set of spatially and possibly temporary distributed rates of extracting water from aquifer (Parelta, 2004).

![Figure (10) Water budget of the entire model at the end of the time step (September 2008).](image-url)
Figure (11) Differences in hydraulic heads between the two simulated stress periods (wet and dry periods)

To predict by simulation the aquifer condition after 20 years, the stress period of this duration should be divided into 40 stress periods: one year simulation is divided into 2 period lengths, for the wet season extending to 210 days and for the dry period extending to 150 days.

A continuous drop in groundwater level was noticed at indifferent locations of the aquifer.

Practical Views about Validation:

Refsgaard (2001) defines model validation as the process of demonstrating that a given site-specific model is capable of making accurate predictions for periods outside the calibration period. He also states that a model is said to be validated if its accuracy and predictive capability in the validation period have been proven to lie within acceptable limits or errors.

To ensure that the model is valid, 4 different measured readings are compared with the calculated hydraulic head by the model for 4 wells, the following figures (12), (13), (14) and (15) show the calculated hydraulic heads for those four wells. The readings were as follows: (214.75, 258.14, 267.14, 205.03) m, while the measured readings were (215.27, 258.3, 268, 205.56) m respectively.

Good harmony was found between measured and calculated hydraulic heads of the aquifer during different assumed time steps and time series of water levels in the modeled domain. This gives confidence in the model which encourages an attempt to make prediction simulation for the next 20 years, Figures (16) and (17).

Taking into consideration that the average recharge is constant while the average discharge is increasing by (-50) m³/day for each cell as a result of the drilling of 50 new wells in the studied area yearly.
Figure (12) Calculated hydraulic head for cell No. (27,16)

Figure (13) Calculated hydraulic head for cell No. (27,25)
Figure (14) Calculated hydraulic head for cell No. (37,29)

Figure (15) Calculated hydraulic head for cell No. (22,19)
Figure (16) Calculated hydraulic head for the year (2008)

Figure (17) the difference in calculated hydraulic heads for the period (2008-2015)
References:


Parelta, R.C., 2004: Optimization modeling for groundwater and conjunctive water policy, Water dynamic laboratory, Utah state university foundation, Utah state university.


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